ELECTRICAL ENGINEERING

CONTROL SYSTEMS

Comprehensive Theory
with Solved Examples *and* Practice Questions

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Email : infomep@madeeasy.in | **Web :** www.madeeasypublications.org

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CONTENTS

Control Systems

CHAPTER 11

Industrial Controllers and Compensators 354-396

CHAPTER 12

State Variable Analysis 397-450

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CHAPTER

1

Introduction

Control System : Control system is that means by which any quantity of interest in a machine, mechanism or other equipment is maintained or altered in accordance with a desired manner.

Control system can also be defined as the combination of elements arranged in a planned manner wherein each element causes an effect to produce a desired output.

Figure shows the general diagrammatic representation of a typical control system. For the automobile driving system, the input (command) signal is the force on the accelerator pedal which through linkages causes the carburettor value to open (close) so as to increase or decrease fuel (liquid form) flow to the engine bring the engine-vehicle speed (controlled variable) to the desired value.

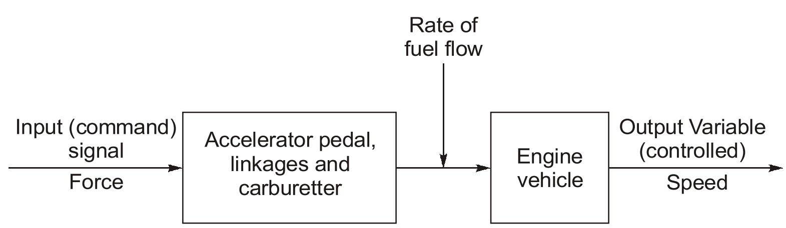


Fig. : *The basic control system*

The diagrammatic representation of above figure is known as block diagram representation wherein each block represents an element, a plant, mechanism, devices etc., whose inner details are not indicated. Each block has an input and output signal which are linked by a relationship characterizing the block. It may be noted that the signal flow through the block is unidirectional.

Control systems are classified into two general categories as Open-loop and closed-loop systems.

1.1 OPEN LOOP CONTROL SYSTEMS

An open loop control system is one in which the control action is independent of the output.

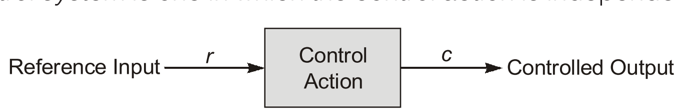


Fig. : *Open-loop control system*

This is the simplest and most economical type of control system and does not have any feedback arrangement.

CHAPTER

2

Transfer Function

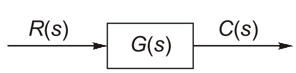
2.1 TRANSFER FUNCTION AND IMPULSE RESPONSE FUNCTION

In control theory, transfer functions are commonly used to characterise the input-output relationships of components or systems that can be described by linear, time-invariant differential equations.

Transfer Function

The transfer function of a linear, time-invariant, differential equation system is defined as the ratio of the Laplace transform of the output (response function) to the Laplace transform of the input (driving function) under the assumption that all initial conditions are zero.

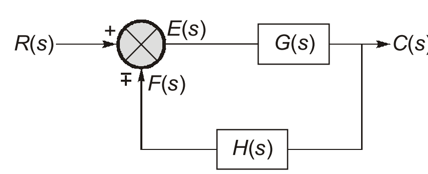
Transfer Function of Open Loop System :



$$G(s) = \frac{C(s)}{R(s)}$$

Transfer Function of Closed Loop System :

Transfer function of closed loop system



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

$R(s)$ = Reference input

$C(s)$ = Controlled output

$E(s)$ = Actuating error signal

$G(s)$ = Forward path transfer function

$H(s)$ = Feedback path transfer function

$$C(s) = G(s)E(s)$$

$$= G(s)[R(s) \pm C(s)H(s)] = G(s)[R(s) \pm G(s)C(s)H(s)$$

$$C(s) \pm G(s)H(s)C(s) = G(s)R(s)$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

Shortcut Method

1. To find close loop transfer function from open loop transfer function

If $\qquad \text{O.L.T.F.} = \dfrac{\text{Numerator}}{\text{Denominator}}$

Then, $\qquad \text{C.L.T.F.} = \dfrac{\text{Numerator}}{\text{Denominator} + \text{Numerator}}$

2. To find open loop transfer function from close loop transfer function

If $\qquad \text{C.L.T.F.} = \dfrac{\text{Numerator}}{\text{Denominator}}$

Then, $\qquad \text{O.L.T.F.} = \dfrac{\text{Numerator}}{\text{Denominator} - \text{Numerator}}$

Linear Systems

A system is called linear if the principle of superposition and principle of homogeneity apply. The principle of superposition states that the response produced by the simultaneous application of two different forcing functions is the sum of the two individual responses. Hence, for the linear system, the response to several inputs can be calculated by transferring one input at a time and adding the results. It is the principle that allows one to build up complicated solutions to the linear differential equations from simple solutions.

In an experimental investigation of a dynamic system, if cause and effect are proportional, thus implying that the principle of superposition holds, then the system can be considered as linear.

Linear Time-Invariant Systems and Linear-Time Varying Systems

A differential equation is linear if the coefficients are constants or functions only of the independent variable. Dynamic systems that are composed of linear time-invariant lumped-parameter components may be described by linear time-invariant differential equations i.e. constant-coefficient differential equations. Such systems are called linear time-invariant (or linear constant-coefficient) systems. Systems that are represented by differential equations whose coefficients are function of time are called linear time varying systems. An example of a time-varying control system is a space craft control system (the mass of a space craft changes due to fuel consumption).

The definition of transfer function is easily extended to a system with multiple inputs and outputs (i.e. a multivariable system). In a multivariable system, a linear differential equation may be used to describe the relationship between a pair of input and output variables, when all other inputs are set to zero. Since the principle of superposition is valid for linear systems, the total effect (on any output) due to all the inputs acting simultaneously is obtained by adding up the outputs due to each input acting alone.

EXAMPLE : 2.1

When deriving the transfer function of a linear element

(a) both initial conditions and loading are taken into account.

(b) initial conditions are taken into account but the element is assumed to be not loaded.

(c) initial conditions are assumed to be zero but loading is taken into account.

(d) initial conditions are assumed to be zero and the element is assumed to be not loaded.

Solution : (c)

While deriving the transfer function of a linear element only initial conditions are assumed to be zero, loading (or input) can't assume to be zero.

Using equation (i) when input is $u(t)$, output is

$$\frac{H(s+c)}{s(s+a)(s+b)} = \frac{K_1}{s} + \frac{D}{s+a} + \frac{E}{s+b}$$

Taking inverse Laplace transform,

$$\text{Output} = 2 + De^{-t} + Ee^{-3t}$$

So, $a = 1$ and $b = 3$

Using final value theorem

$$\Rightarrow \quad \lim_{s\to 0} \frac{s \cdot H(s+c)}{s(s+a)(s+b)} = \lim_{t\to\infty} 2 + De^{-t} + Ee^{-3t}$$

$$\frac{Hc}{ab} = 2 \quad \text{and} \quad Hc = 6$$

Using equation (i) when input is $e^{-2t}u(t)$, output is $\dfrac{H(s+c)}{(s+2)(s+a)(s+b)}$

Only two terms are present in the response.

Hence $s + c = s + 2$

$\Rightarrow$ $c = 2$

$H = 3$ $\qquad (\because HC = 6)$

OBJECTIVE BRAIN TEASERS

Q.1 A control system with certain excitation is governed by the following mathematical equation $\dfrac{d^2x}{dt^2} + \dfrac{1}{2}\dfrac{dx}{dt} + \dfrac{1}{18}x = 10 + 15e^{-4t} + 2e^{-5t}$. The natural time constants of the response of the system are

(a) $2s$ and $5s$ (b) $3s$ and $6s$

(c) $4s$ and $5s$ (d) $\frac{1}{3}s$ and $\frac{1}{6}s$

Q.2 The response $g(t)$ of a linear time invariant system to an impulse $\delta(t)$, under initially relaxed condition is $g(t) = e^{-t} + e^{-2t}$. The response of this system for a unit step input $u(t)$ is

(a) $(1 + e^{-t} + e^{-2t})u(t)$

(b) $(e^{-t} + e^{-2t})u(t)$

(c) $(1.5 - e^{-t} - 0.5e^{-2t})u(t)$

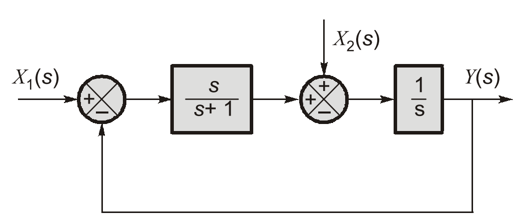
(d) $e^{-t}\delta(t) + e^{-2t}u(t)$

Q.3 The frequency response of a linear time-invariant system is given by $H(f) = \dfrac{5}{1 + j10\pi f}$. The step response of the system is

(a) $5(1 - e^{-5t})u(t)$ (b) $5\left(1 - e^{-t/5}\right)u(t)$

(c) $\frac{1}{5}\left(1 - e^{-5t}\right)u(t)$ (d) $\dfrac{1}{(s+5)(s+1)}$

Q.4 For the following system :



when $X_1(s) = 0$, the transfer function $\dfrac{Y(s)}{X_2(s)}$ is

(a) $\dfrac{s+1}{s^2}$ (b) $\dfrac{1}{s+1}$

(c) $\dfrac{s+2}{s(s+1)}$ (d) $\dfrac{s+1}{s(s+2)}$

Q.14 The transfer function of a system is given by $\frac{C(s)}{R(s)} = \frac{100}{(s+10)(s^2+2s+1)}$, using the concept of dominant pole, the 2nd order approximation of above transfer function is

(a) $\frac{100}{s^2+2s+1}$ (b) $\frac{10}{s^2+2s+1}$

(c) $\frac{10}{s+10}$ (d) $\frac{100}{(s+10)}$

Q.15 A differentiator has a transfer function whose

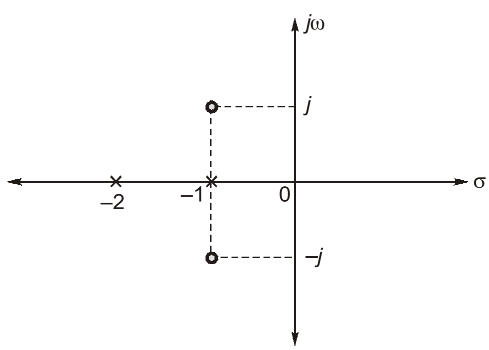
(a) Magnitude decreases linearly with frequency

(b) Magnitude increases linearly with frequency

(c) Phase increases linearly with frequency

(d) Phase is constant

Q.16 The pole-zero plot of a system is given below. If $G(s) = 15$ for $s = 2$, then the transfer function of the system is



(a) $\frac{12(s^2+2s+5)}{(s+1)(s+2)}$ (b) $\frac{18(s^2+2s+2)}{(s+1)(s+2)}$

(c) $\frac{12(s^2+2s+3)}{(s+1)(s+2)}$ (d) $\frac{6(s^2+2s+3)}{(s+1)(s+2)}$

Q.17 A linear time invariant system initially at rest, when subjected to unit step input gives a response of $2te^{-5t}$, $t > 0$, the corresponding transfer function is

(a) $\frac{2}{s(s+5)^2}$ (b) $\frac{2s}{(s+5)}$

(c) $\frac{2s}{(s+5)^2}$ (d) $\frac{2s}{(s-5)^2}$

ANSWER KEY

1. (b)	**2.** (c)	**3.** (b)	**4.** (d)	**5.** (a)
6. (d)	**7.** (d)	**8.** (d)	**9.** (c)	**10.** (d)
11. (c)	**12.** (b)	**13.** (b)	**14.** (b)	**15.** (b, d)
16. (b)	**17.** (c)			

HINTS & EXPLANATIONS

1. (b)

Natural time constants of the response depend only on poles of the system.

$$T(s) = \frac{C(s)}{R(s)} = \frac{1}{s^2 + s/2 + 1/18} = \frac{18}{18s^2 + 9s + 1} = \frac{1}{(6s+1)(3s+1)}$$

This is in the form $\frac{1}{(1+sT_1)(1+sT_2)}$

$\therefore$ $T_1, T_2 = 6$ sec, 3 sec.

2. (c)

Transfer function of system is impulse response of the system with zero initial conditions.

Transfer function $= G(s) = \mathcal{L}\left(e^{-t} + e^{-2t}\right)$

$$= \frac{1}{s+1} + \frac{1}{s+2}$$

$$G(s) = \frac{C(s)}{R(s)} = \left(\frac{1}{s+1} + \frac{1}{s+2}\right)$$

For step input, $R(s) = \frac{1}{s}$

$$C(s) = R(s)\cdot G(s) = \frac{1}{s}\left(\frac{1}{s+1} + \frac{1}{s+2}\right) = \frac{1}{s(s+1)} + \frac{1}{s(s+2)}$$

$$C(s) = \left(\frac{1}{s} - \frac{1}{s+1}\right) + \frac{1}{2}\left(\frac{1}{s} - \frac{1}{s+2}\right)$$

$$= \frac{1.5}{s} - \frac{1}{s+1} - \frac{0.5}{s+2}$$

$$\text{Response} = c(t) = \mathcal{L}^{-1}\left[C(s)\right]$$

$$= \mathcal{L}^{-1}\left[\frac{1.5}{s} - \frac{1}{s+1} - \frac{0.5}{s+2}\right]$$

$$c(t) = (1.5 - e^{-t} - 0.5e^{-2t})u(t)$$

3. (b)

$$H(f) = \frac{5}{1 + j10\pi f}$$

$$H(s) = \frac{5}{1+5s} = \frac{5}{5\left(s+\frac{1}{5}\right)} = \frac{1}{s+\frac{1}{5}}$$

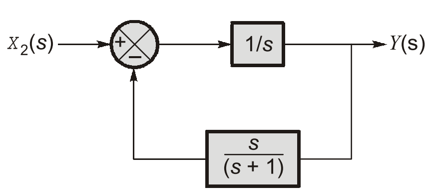
$$\text{Step response} = \frac{1}{s}\frac{1}{\left(s+\frac{1}{5}\right)}$$

$$Y(s) = \frac{5}{s} - \frac{5}{s+\frac{1}{5}}$$

$$\Rightarrow \quad y(t) = 5[1 - e^{-t/5}]\, u(t)$$

4. (d)

Redrawing the block diagram with $X_1(s) = 0$



The transfer function

$$T(s) = \frac{Y(s)}{X_2(s)} = \frac{G(s)}{1+G(s)H(s)} \qquad \text{...(i)}$$

Here, $G(s) = \frac{1}{s}$ and $H(s) = \frac{s}{s+1}$

$$\frac{Y(s)}{X_2(s)} = \frac{1/s}{1+\frac{1}{s}\times\frac{s}{s+1}} = \frac{(s+1)}{s(s+2)}$$

5. (a)

$$\frac{C(s)}{R(s)} = \frac{s+1}{s+2}$$

$$\therefore \quad C(s) = R(s)\cdot\frac{s+1}{s+2}$$

$$= \frac{1}{s^2}\cdot\frac{s+1}{s+2} = \frac{1}{s^2}\left(1 - \frac{1}{s+2}\right)$$

$$= \frac{1}{s^2} - \frac{1}{s^2(s+2)}$$

$$\frac{1}{s^2}\cdot\left(\frac{s+1}{s+2}\right) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2}$$

$$s + 1 = As(s+2) + B(s+2) + Cs^2$$

$$= As^2 + 2As + Bs + 2B + Cs^2$$

$$\therefore \quad A + C = 0,\ 2A + B = 1 \text{ and } 2B = 1$$

$$\therefore \quad A = \frac{1}{2},\quad B = \frac{1}{4},\quad C = -\frac{1}{4}$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{1}{4s} + \frac{1}{2s^2} + \left(-\frac{1}{4}\right)\frac{1}{s+2}$$

$$= \frac{1}{4}u(t) + \frac{1}{2}tu(t) - \frac{1}{4}e^{-2t}u(t)$$

6. (d)

(i) Transfer function can be obtained from signal flow graph of the system.

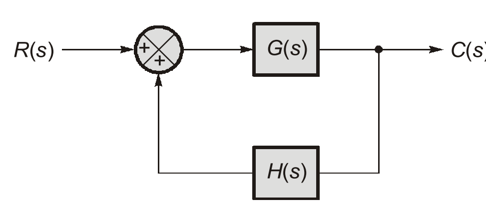
(ii) Transfer function typically characterizes to LTI systems.

(iii) Transfer function gives the ratio of output to input in s-domain of system.

$$\text{TF} = \left.\frac{L[\text{Output}]}{L[\text{Input}]}\right|_{\text{Initial conditions} = 0}$$

8. (d)

Block diagram of regenerating feedback system



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1-G(s)H(s)}$$

CHAPTER

3

Block Diagrams

3.1 BLOCK DIAGRAMS : FUNDAMENTALS

A block diagram of a system is a pictorial representation of the functions performed by each component and of the flow of signals. Such a diagram depicts the interrelationships that exist among the various components. Differing from a purely abstract mathematical representation, a block diagram has the advantage of indicating more realistically the signal flows of the actual system.

In a block diagram, all system variables are linked to each other through functional blocks. The **functional block** or simple **block** is a symbol for the mathematical operation on the input signal to the block that produces the output. The transfer functions of the components are usually entered in the corresponding blocks, which are connected by arrows to indicate the direction of the flow of signals. Note that the signal can pass only in the direction of the arrows. Thus, a block diagram of a control system explicitly shows a unilateral property.

***Fig. :** Element of a block diagram*

$$C(s) = R(s) \cdot G(s)$$

$$G(s) = \frac{C(s)}{R(s)}$$

Note: The dimension of the output signal from the block is the dimension of the input signal multiplied by the dimension of the transfer function in the block.

The advantages of the block diagram representation of a system are that it is easy to form the overall block diagram for the entire system by merely connecting the blocks of the components according to the signal flow and it is possible to evaluate the contribution of each component to the overall performance of the system. If the mathematical and functional relationships of all the system elements are known, the block diagram can be used as a tool for the analytic or computer solution of the system. In general, block diagrams can be used to model linear as well as non-linear systems.

It should be noted that the block diagram of a given system is not unique. A number of different block diagrams can be drawn for a system, depending on the point of view of the analysis.

Take off Point or Branch Point

A take off point is a point from which the signal from a block goes concurrently to other blocks or summing points. It should be noted that such taking off from any signal does not alter the parent signal. This permits the signal to proceed unaltered along several different paths to several destinations.

Summing Point

Summing points are used to add two or more signals in the system. A circle with a cross, is the symbol that indicates a summing point. The plus or minus sign at each arrowhead indicates whether that signal is to be added or subtracted.

Constructing Block Diagram for Control System

A control system can be represented diagramatically by block diagram. The differential equations governing the system are used to construct the block diagram. By taking Laplace transform the differential equations are converted to algebraic equations. Th equations will have variables and constants. From the working knowledge of the system, the input and output variables are identified and the block diagram for each equation can be drawn. Each equation gives one section of block diagram. The output of the one section will be input for another section. The various sections are interconnected to obtain the overall block diagram of the system.

3.2 BLOCK DIAGRAM OF A CLOSED-LOOP SYSTEM

Figure shows the block diagram of a negative feedback system. With reference to this figure, the terminology used in block diagrams of control systems is given below.

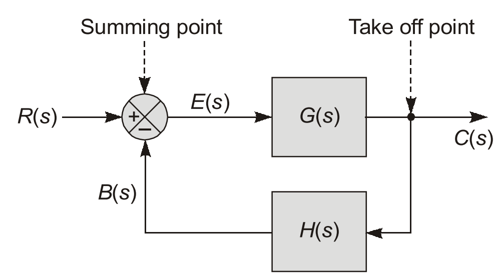


Fig. : *Block diagram of closed-loop system*

$R(s)$ = reference input signal

$C(s)$ = output signal or controlled variable

$B(s)$ = feedback signal

$E(s)$ = actuating signal or error signal

$$G(s) = \frac{C(s)}{E(s)} = \text{forward path transfer function}$$

$H(s)$ = feedback path transfer functions

$$G(s)H(s) = \frac{B(s)}{E(s)} = \text{open-loop transfer function}$$

$$T(s) = \frac{C(s)}{R(s)} = \text{closed-loop transfer function}$$

$$\frac{E(s)}{R(s)} = \text{Error ratio}$$

$$\frac{B(s)}{R(s)} = \text{Primary feedback ratio}$$

From figure,

$$C(s) = E(s)G(s) \quad \text{...(i)}$$

$$E(s) = R(s) - B(s) = R(s) - H(s)\,C(s) \quad \text{...(ii)}$$